Technical Appendix

Digital Media Mergers: Theory and Application to Facebook-Instagram

Justin Katz and Hunt Allcott

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G Additional Model Derivations

G.1 Reaction Functions

This appendix presents additional intuition on strategic incentives as the two platforms compete. We do this primarily with reaction functions computed numerically at example parameter values. To simplify the analysis, we restrict the model from the main text in two ways. First, we assume time on platform $\boldsymbol{T}_i$ is exogenous but not necessarily homogeneous. Second, we assume $\zeta_j = 1$, so that duplicated impressions are fully wasted, and that $\boldsymbol{c} = \mathbf{0}$. We first present the first-order conditions before illustrating them using numerical examples. Derivations are in Appendix G.2.

G.1.1 First-Order Conditions

Re-arranging equation (13), the merged platform chooses ad load to satisfy

$$\alpha_j^{e,m} = \frac{\sum_i \alpha_{-j} \cdot T_{i,-j} \cdot \frac{\partial p_i}{\partial \alpha_j} + T_{ij} \cdot p_i}{-\sum_i T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_j}} \tag{T1}$$

$$\text{where } \frac{\partial p_i}{\partial \alpha_j} = -\frac{\eta}{Am} \cdot T_{ij}.$$

Rearranging equation (16), the separated platforms choose ad load to satisfy

$$\alpha_j^{e,s} = \frac{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot p_{ij}}{-\sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial p_{ij}}{\partial \alpha_j}}. \tag{T2}$$

G.1.2 Numerical Examples

We explore how the above forces impact ad load in the separated relative to combined equilibrium through several numerical examples. We parameterize the distribution of time use with a measure M of multi-homers with $\boldsymbol{T}_{mi} \sim \mathcal{N}\left(\frac{\mathbf{1}}{\mathbf{2}}, \boldsymbol{\Sigma}_T\right)$, and a measure N_j of single-homers $T_{si} = 1$. Unless otherwise specified, we set $\zeta_j = m = 1$ and the remaining model parameters to their estimated value from Section 4.

Example 1: Partial user overlap (with coordinated ad delivery). We first focus on how partial user overlap impacts the magnitude of the advertiser-side Cournot externality when separated platforms can coordinate to avoid duplication. We set $\boldsymbol{\Sigma}_T = 0$, so that multi-homers split one unit of time equally across platforms.

The merged equilibrium solution is standard linear Cournot: $\alpha_j^{e,m} = \frac{1}{2} Am \cdot (1 + \eta_0)$. In separated equilibrium, reaction functions are:

$$\alpha_j^{e,s,i}(\alpha_{-j}) = \frac{2 - \mu_j}{(4 - 3\mu_j)} Am \cdot (1 + \eta_0) - \frac{1}{2} \frac{\mu_j}{(4 - 3\mu_j)} \alpha_{-j}. \tag{T3}$$

When there is no overlap ($\mu_j = 0$), each separated platform behaves as a monopolist. As overlap increases, separated platforms respond more to their rival's actions, as they have a greater impact on revenue, and internalize less the price impact of increased ad load. Appendix Section G.2.1 derives a closed-form expression for separated equilibrium ad load, and shows that the percent change relative to the merged equilibrium *only* depends on overlap statistics and is independent of ad demand parameters.

Figure T1, panel (a) plots reaction functions and equilibria in to show these forces concretely. Facebook reaction functions to Instagram ad load are plotted horizontally, and Instagram reaction functions to Facebook ad load are plotted vertically. Equilibria are plotted with black dots where the reaction functions intersect. Black lines plot vertical reaction functions when $\mu_j = 0$, because platforms ignore their rival's actions. The blue and orange solid lines plot reaction functions when $\mu_j = 1$ for both platforms, indicating a 40% increase in ad load relative to the merged equilibrium. The dashed lines plot reaction functions given empirically-observed μ_j, with $\mu_{IG} > \mu_{FB}$. Since Instagram has more overlap than Facebook, it will internalize less of the impact of increased ad load on equilibrium prices, and hence increase ad load by much more than Facebook.

Example 2: Uncoordinated ad delivery (with full user overlap). We next consider a separated equilibrium where platforms cannot coordinate to avoid duplication. To focus on the role of duplication, we assume all users are multi-homers with heterogeneous time use parameterized by $\boldsymbol{\Sigma}_T = \text{diag}\left(\sigma^2, \sigma^2\right)$. Heterogeneous time use ensures that the marginal duplication function is continuous and differentiable.

Reaction functions no longer have a closed form, but are given by

$$\alpha_j^{e,s,i}(\alpha_{-j}) = \arg\max_{\alpha_j} \left(1 - O'_{aj}\right) \cdot \eta \left(\sum_i \alpha_j T_{ij} \cdot \left(1 + \eta_0 - \frac{\alpha_j T_{ij}}{Am}\right)\right), \tag{T4}$$

This expression comes from substituting $c_j = 0$, $T_{ij}(\boldsymbol{\alpha}) = T_{ij}$, and the formula for separated equilibrium prices in equation (8) into problem (15), where O'_{aj} is given by equation (9). Rival ad load α_{-j} only enters the expression through marginal duplication O'_{aj}. This makes clear how duplication affects revenue from ad prices directly and strategically through the business stealing effect. Moreover, the variance of time use, controlled by σ^2, will alter separated platform incentives. When σ^2 is very low, platforms can precisely affect marginal duplication and user prices through changes in ad load, amplifying strategic differences relative to the merged equilibrium. When σ^2 is very high, changes in ad load have little impact on marginal duplication, dampening differences relative to the merged equilibrium.

Figure T1, panel (b) illustrates these points. The solid blue and orange lines plot Facebook and Instagram reaction functions when σ^2 is relatively low. Ad load increases significantly. Because platforms have fine control over marginal duplication, the business stealing effect is strong, making choice of ad load a strategic complement. The dashed lines plot reaction functions when σ^2 is high. Ad load still shifts out due to the direct effect. However, platforms have much less control over marginal duplication through choice of ad load, severely dampening the business stealing effect.

Example 3: Uncoordinated ad delivery and partial user overlap together. Finally, we consider the impact of uncoordinated ad delivery and partial user overlap jointly. Reaction functions are the same as in equation (T4). However, the value of O'_{aj} changes because O'_{aj} is lower when a lower fraction of users are multi-homers. To see this, note that:

$$O'_{aj} = \frac{\sum_{i \in \mathcal{U}_m} \mathbf{1}\left[\alpha_j T_{ij} \leq \alpha_{-j} T_{i,-j}\right]}{N_j} \tag{T5}$$

$$= \mu_j \cdot \Pr\left(\alpha_j T_{ij} \leq \alpha_{-j} T_{i,-j} | i \in \mathcal{U}_m\right). \tag{T6}$$

The first line follows because $T_{i,-j} = 0$ implies the numerator is zero for $i \notin \mathcal{U}_m$. The second line means that a lower fraction of multi-homers, and hence lower μ_j, implies lower marginal overlap, holding fixed $\boldsymbol{\alpha}$ and the time use distribution.

Figure T1, panel (c) plots the resulting reaction functions, where all reaction functions use the empirical overlap μ_j on Facebook and Instagram. First, partial overlap ($\mu_j < 1$) dampens the impact of duplication, since each platform has a user population for whom strategic incentives do not change relative to the merged equilibrium. Second, the business stealing motive appears stronger for Instagram than Facebook. This is because Instagram has greater overlap, magnifying the returns to business stealing. Finally, higher time use variance again dampens the business stealing effect, although to a lesser degree as it is already dampened by overlap.

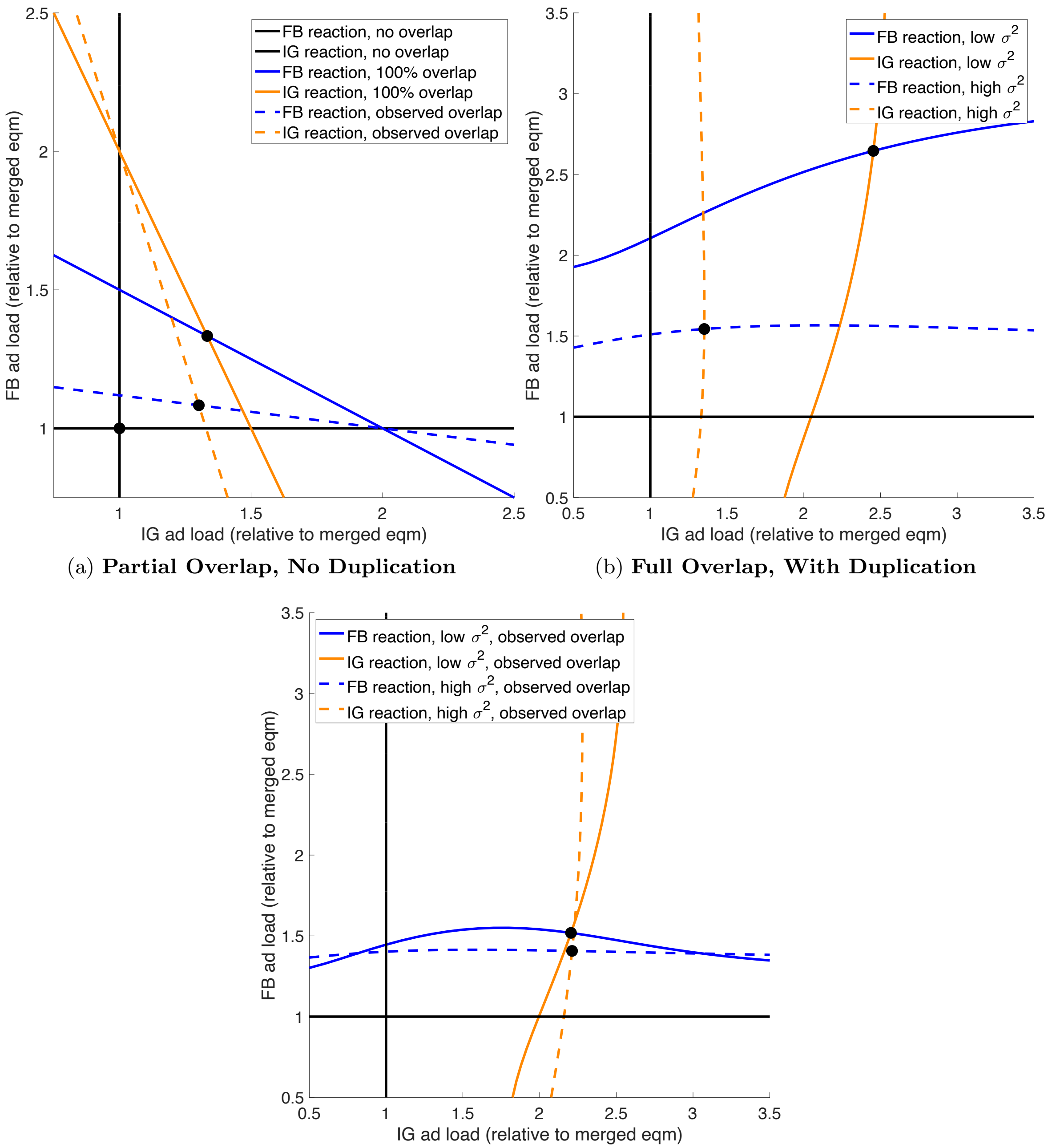
Summary of numerical examples. With exogenous but heterogeneous time use, separating platforms leads to a greater increase in ad load when user overlap across platforms is greater. First, higher overlap leads merged platforms to constrain ad load by more relative to separated platforms, which do not consider the impact of lower ad prices on their rival's revenue. Second, if platforms can't coordinate to avoid inefficient duplication, ad load increases by even more in the separated equilibrium due to the "business stealing" incentive to increase ad load and reduce marginal duplication.

G.1.3 Possibility of Strategic Complementarity

Section G.1.2 gives examples where business stealing changes ad load from strategic substitutes into strategic complements: reaction functions are globally downwards-sloping in Figure T1, panel (a) with coordinated ad delivery, but can be upwards sloping in panel (c) with uncoordinated ad delivery. The following proposition shows formally that optimal ad loads are always strategic substitutes when ad delivery is coordinated but may be strategic complements when ad delivery is uncoordinated.

Proposition 1 (Advertiser-side strategic complementarity)**.** *Suppose that time use $\boldsymbol{T}_i$ is exogenous with $\frac{\partial \boldsymbol{T}_i}{\partial \alpha_j} = \mathbf{0}$ and a well-defined distribution for $\frac{T_{ij}}{T_{i,-j}}$, O'_{aj} is twice differentiable, Assumptions 1 and 2 hold, $\mathbf{c} = \mathbf{0}$, and $\zeta_j = 1$. Then in the separated equilibrium in Problem (15), ad load choices are strategic substitutes if ad delivery is coordinated, and either strategic complements or substitutes if ad delivery is uncoordinated.*

Figure T1: **Key Advertiser-Side Forces: Numerical Examples**



(a) **Partial Overlap, No Duplication** (b) **Full Overlap, With Duplication**

(c) **Partial Overlap and Duplication Together**

Notes: This figure presents numerical examples described in Section G.1.2. Panel (a) plots reaction functions in an example with partial overlap but no duplication in separated equilibrium. Panel (b) plots reaction functions in an example with full overlap and duplication. Panel (c) plots reaction functions in an example with partial overlap and duplication.

G.2 Derivations for Section G.1

This subsection derives expressions in Section G.1.

Derivation of equation (T1). The merged firm's problem is

$$\boldsymbol{\alpha}^{e,m} = \arg\max_{\boldsymbol{\alpha}} \sum_i p_i \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i, \tag{T7}$$

where the only difference relative to equation (12) is that time use does not depend on $\boldsymbol{\alpha}$ as it is exogenous and $\boldsymbol{c} = \mathbf{0}$. The first-order conditions are

$$\begin{aligned} 0 &= \frac{\partial}{\partial \alpha_j}\left[\sum_i p_i \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i\right] \\ &= \sum_i \boldsymbol{\alpha} \cdot \boldsymbol{T}_i \cdot \frac{\partial p_i}{\partial \alpha_j} + p_i \cdot T_{ij}. \end{aligned} \tag{T8}$$

Rearranging:

$$\alpha_j = -\frac{\sum_i \alpha_{-j} \cdot T_{i,-j} \cdot \frac{\partial p_i}{\partial \alpha_j} + T_{ij} \cdot p_i}{\sum_i T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_j}} \quad \text{where } \frac{\partial p_i}{\partial \alpha_j} = -\frac{\eta}{Am} \cdot T_{ij}, \tag{T9}$$

and where the value of $\frac{\partial p_i}{\partial \alpha_j}$ comes from differentiating equation (4) with respect to α_j and applying the inverse function rule. □

Derivation of equation (T2). The separated platform problem is

$$\alpha_j^{e,s,i} = \arg\max_{\alpha_j} \sum_{i \in \mathcal{U}_j} \alpha_j \cdot T_{ij} \cdot p_{ij} \tag{T10}$$

The first-order condition is

$$0 = \sum_{i \in \mathcal{U}_j} \alpha_j \cdot T_{ij} \cdot \frac{\partial p_{ij}}{\partial \alpha_j} + T_{ij} \cdot p_{ij}. \tag{T11}$$

Rearranging gives the expression:

$$\alpha_j = -\frac{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot p_{ij}}{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_j}}, \tag{T12}$$

where

$$\frac{\partial p_{ij}}{\partial \alpha_j} = -\left(1 - O'_{aj}\right) \cdot \frac{\eta}{Am} \cdot T_{ij} - \frac{p_{ij}}{\left(1 - O'_{aj}\right)} \frac{\partial O'_{aj}}{\partial \alpha_j}, \tag{T13}$$

which gives the expressions in the text. □

Proof of Proposition 1. To show ad load choices are strategic substitutes in Problem (16) with coordinated ad delivery, first note that given Assumptions 1 and 2,

$$\frac{\partial p_i}{\partial \alpha_j} = -\frac{\eta}{Am} \cdot T_{ij} \implies \frac{\partial^2 p_i}{\partial \alpha_{-j} \partial \alpha_j} = 0. \tag{T14}$$

Differentiate equation (T2) with respect to α_{-j} and substitute $p_{ij} = p_i$ to reflect coordinated ad delivery:

$$\frac{\partial \alpha_j^{e,s,i}}{\partial \alpha_{-j}} = -\frac{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_{-j}}}{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_j}} = -\frac{\sum_{i \in \mathcal{U}_j} T_{ij} T_{i,-j}}{\sum_{i \in \mathcal{U}_j} T_{ij}^2}. \tag{T15}$$

This proves that $\frac{\partial \alpha_j^{e,s,i}}{\partial \alpha_{-j}} \leq 0$, meaning choices of ad load are strategic substitutes.

To show that ad load choices can be strategic complements in Problem (15) with uncoordinated ad delivery, define the distribution of $T_{ij}/T_{i,-j}$ as $\mathcal{T}_{-j}$ and note that

$$O'_{aj} = \Pr\left(\alpha_j T_{ij} \leq \alpha_{-j} T_{i,-j}\right) = \mathcal{T}_{-j}\left(\alpha_{-j}/\alpha_j\right). \tag{T16}$$

Therefore,

$$\frac{\partial O'_{aj}}{\partial \alpha_{-j}} = \mathcal{T}'_{-j}\left(\frac{\alpha_{-j}}{\alpha_j}\right) \cdot \alpha_j^{-1} \geq 0, \quad \frac{\partial O'_{aj}}{\partial \alpha_j} = -\mathcal{T}'_{-j}\left(\frac{\alpha_{-j}}{\alpha_j}\right) \cdot \frac{\alpha_{-j}}{\alpha_j^2} \leq 0. \tag{T17}$$

Moreover,

$$\frac{\partial O'_{aj}}{\partial \alpha_{-j} \partial \alpha_j} = -\mathcal{T}'_{-j}\left(\frac{\alpha_{-j}}{\alpha_j}\right) \cdot \frac{1}{\alpha_j^2} - \frac{\alpha_{-j}}{\alpha_j^3} \cdot \mathcal{T}''_{-j}\left(\frac{\alpha_{-j}}{\alpha_j}\right), \tag{T18}$$

which has ambiguous sign. Differentiate equation (T2) with respect to α_{-j} in the case with uncoordinated ad delivery (with p_{ij} separate on each platform and given by equation (8)):

$$\frac{\partial \alpha_j^{e,s,i}}{\partial \alpha_{-j}} = -\frac{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot p_{ij}}{\left(\sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial p_{ij}}{\partial \alpha_j}\right)^2} \cdot \sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial^2 p_{ij}}{\partial \alpha_{-j} \partial \alpha_j}. \tag{T19}$$

Strategic complementarity occurs when $\frac{\partial^2 p_{ij}}{\partial \alpha_{-j} \partial \alpha_j} < 0$, and strategic substitutes when $\frac{\partial^2 p_{ij}}{\partial \alpha_{-j} \partial \alpha_j} > 0$. Moreover,

$$\frac{\partial^2 p_{ij}}{\partial \alpha_{-j} \partial \alpha_j} = \frac{\partial O'_{aj}}{\partial \alpha_{-j}} \cdot \frac{\eta}{Am} T_{ij} + \frac{p_{ij}}{\left(1 - O'_{aj}\right)^2} \frac{\partial O'_{aj}}{\partial \alpha_{-j}} \frac{\partial O'_{aj}}{\partial \alpha_j} - \frac{p_{ij}}{\left(1 - O'_{aj}(\boldsymbol{\alpha}^{e,s,d}\right)} \frac{\partial^2 O'_{aj}}{\partial \alpha_{-j} \partial \alpha_j}. \tag{T20}$$

The first term is positive because $\frac{\partial O'_{aj}}{\partial \alpha_{-j}}$ is. The second term is negative because $\frac{\partial O'_{aj}}{\partial \alpha_j}$ is. The third term has a sign depending on the sign of $\frac{\partial^2 O'_{aj}}{\partial \alpha_{-j} \partial \alpha_j}$, which is ambiguous and depends on the shape of the pdf of $T_{ij}/T_{i,-j}$. Overall, it is thus possible that $\frac{\partial^2 p_{ij}}{\partial \alpha_{-j} \partial \alpha_j} < 0$, implying that $\frac{\partial \alpha_j^{e,s,i}}{\partial \alpha_{-j}} < 0$ (making ad load choices strategic complements). □

Remark. Proposition 1 also holds with arbitrary $\boldsymbol{c}$ and ζ_i that satisfy Assumption 3, where the proof focuses on the restricted case for notational clarity.

G.2.1 Derivations for Section G.1.2

This subsection derives expressions in Section G.1.2. Throughout, we use the notation M and N_j to refer to the number of multi-homers and users on platform j, respectively, satisfying $\mu_j = M/N_j$ and $M + \sum_j (N_j - M) = N$.

Derivation of merged equilibrium ad load. The merged firm's revenue is

$$R^m(\boldsymbol{\alpha}) = \eta \cdot \left(\frac{M}{2} (\alpha_1 + \alpha_2) \cdot \left(1 + \eta_0 - \frac{\alpha_1 + \alpha_2}{2Am}\right) + \sum_j (N_j - M)\alpha_j \left(1 + \eta_0 - \frac{\alpha_j}{Am}\right) \right). \quad \text{(T21)}$$

The first-order condition with respect to α_j is

$$\begin{aligned} 0 &= \frac{M}{2} \left(\left(1 + \eta_0 - \frac{\alpha_j + \alpha_{-j}}{Am}\right) + (N_j - M) \left(1 + \eta_0 - 2\frac{\alpha_j}{Am}\right) \right) \\ &= \frac{\alpha_j}{Am} (3M - 4N_j) + (1 + \eta_0)(2N_j - M) - \frac{O}{A}\alpha_{-j}. \end{aligned} \quad \text{(T22)}$$

Therefore,

$$\alpha_j = \frac{2N_j - M}{4N_j - 3M} Am(1 + \eta_0) - \frac{M}{4N_j - 3M}\alpha_{-j}. \quad \text{(T23)}$$

Substituting in for α_{-j} and simplifying gives

$$\alpha_j = \left(\frac{(2N_j - M)(4N_{-j} - 3M) - M(2N_{-j} - M)}{(4N_j - 3M) \cdot (4N_{-j} - 3M) - M^2} \right) Am(1 + \eta_0). \quad \text{(T24)}$$

The numerator simplifies to $8N_jN_{-j} - 6M(N_j + N_{-j}) + 4M^2$ and the denominator simplifies to $2 \cdot \left(8N_jN_{-j} - 6M(N_j + N_{-j}) + 4M^2\right)$. Therefore, the expression becomes:

$$\alpha_j^{e,m} = \frac{1}{2} Am(1 + \eta_0), \quad \text{(T25)}$$

as reported in the text. □

Derivation of Equation (T3). Separated platform revenue is

$$R_j^s(\boldsymbol{\alpha}) = \eta \cdot \left(\frac{M}{2}\alpha_j \cdot \left(1 + \eta_0 - \frac{\alpha_1 + \alpha_2}{2Am}\right) + (N_j - M)\alpha_j \left(1 + \eta_0 - \frac{\alpha_j}{Am}\right) \right). \quad \text{(T26)}$$

The first-order condition is

$$\begin{aligned} 0 &= \frac{M}{2} \left(1 + \eta_0 - \frac{\alpha_j + \alpha_{-j}}{2Am} - \frac{\alpha_j}{2Am}\right) + (N_j - M)\left(1 + \eta_0 - 2\frac{\alpha_j}{Am}\right) \\ &= \frac{\alpha_j}{Am} (3M - 4N_j) + (1 + \eta_0)(2N_j - M) - \frac{M}{Am}\alpha_{-j}. \end{aligned} \quad \text{(T27)}$$

Therefore,

$$\alpha_j^{e,s,i}(\alpha_{-j}) = \frac{2N_j - M}{4N_j - 3M} Am(1+\eta_0) - \frac{M}{(4N_j - 3M)}\alpha_{-j}$$
$$= \frac{2-\mu_j}{4-3\mu_j} Am(1+\eta_0) - \frac{\mu_j}{4-3\mu_j}\alpha_{-j}, \tag{T28}$$

where we factor out N_j from the numerator and denominator to express in terms of μ_j. Substituting for α_{-j} and simplifying yields:

$$\alpha_j^{e,s,i} = \left(\frac{(2N_j - M)(4N_{-j} - 3M) - O(2N_{-j} - M)}{(4N_j - 3M)\cdot(4N_{-j} - 3M) - M^2}\right) Am(1+\eta_0). \tag{T29}$$

□

Percent change in ad load only depends on overlap statistics. Using equations (T25) and (T29), the percent increase in ad load in the separated equilibrium is

$$\frac{\alpha_j^{e,s,i}}{\alpha_j^{e,m}} - 1 = 2\cdot\left(\frac{(2N_j - M)(4N_{-j} - 3M) - M(2N_{-j} - M)}{(4N_j - 3M)\cdot(4N_{-j} - 3M) - M^2}\right) - 1$$
$$= 2\cdot\left(\frac{(2-\mu_j)(4-3\mu_{-j}) - \mu_j(2-\mu_{-j})}{(4-3\mu_j)\cdot(4-3\mu_{-j}) - \mu_j\mu_{-j}}\right) - 1. \tag{T30}$$

This only depends on overlap statistics, proving the claim in the text. □

G.3 Derivations for Section 1.3

This section analyzes the full model under various ownership structures. Throughout, we will apply assumptions 1 and 2.

G.3.1 Social Planner Benchmark

The analysis follows each point in Section 1.3.1. The planner chooses

$$\boldsymbol{\alpha}^o = \arg\max_{\boldsymbol{\alpha}} \sum_i U_i^*\left(\boldsymbol{T}(\boldsymbol{\alpha}), n; \boldsymbol{\alpha}\right) + \sum_i Am \cdot \int_{x=p_i(\boldsymbol{\alpha})}^{(\pi\omega)^{\max}} x dH(x) - \boldsymbol{c}\cdot\boldsymbol{\alpha}. \tag{T31}$$

The first-order condition is

$$0 = \sum_i \frac{\partial U_i^*(\cdot;\boldsymbol{\alpha})}{\partial \alpha_j} - \sum_i \frac{Am}{\eta}\cdot p_i(\boldsymbol{\alpha})\cdot\frac{\partial p_i(\boldsymbol{\alpha})}{\partial \alpha_j} - c_j. \tag{T32}$$

Since

$$\frac{\partial p_i(\boldsymbol{\alpha})}{\partial \alpha_j} = -\frac{\eta}{Am}\cdot\left[T_{ij} + \alpha_j\frac{\partial T_{ij}}{\partial \alpha_j} + \alpha_{-j}\frac{\partial T_{i,-j}}{\partial \alpha_j}\right], \tag{T33}$$

the FOC becomes

$$0 = \sum_i \frac{\partial U_i^*(\cdot;\boldsymbol{\alpha})}{\partial \alpha_j} - \sum_i p_i(\boldsymbol{\alpha}) \cdot \left[T_{ij} + \alpha_j \frac{\partial T_{ij}}{\partial \alpha_j} + \alpha_{-j} \frac{\partial T_{i,-j}}{\partial \alpha_j} \right] - c_j. \tag{T34}$$

Solving for α_j gives

$$\alpha_j^o = \frac{\sum_i \frac{\partial U_i^*(\cdot;\boldsymbol{\alpha})}{\partial \alpha_j} + \sum_i p_i(\boldsymbol{\alpha}) \cdot \left(T_{ij} + \alpha_{-j} \frac{\partial T_{i,-j}}{\partial \alpha_j} \right) - c_j}{- \sum_i p_i(\boldsymbol{\alpha}) \cdot \frac{\partial T_{ij}}{\partial \alpha_j}}. \tag{T35}$$

G.3.2 Merged Platform Solution

The problem is:

$$\max_{\boldsymbol{\alpha}} \sum_i p_i(\boldsymbol{\alpha}) \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) - \boldsymbol{c} \cdot \boldsymbol{\alpha}. \tag{T36}$$

The first-order conditions are

$$0 = \sum_i \left[\frac{\partial p_i}{\partial \alpha_j} \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) + p_i(\boldsymbol{\alpha}) \cdot T_{ij}(\boldsymbol{\alpha}) + p_i(\boldsymbol{\alpha}) \cdot \boldsymbol{\alpha} \cdot \frac{\partial \boldsymbol{T}_i(\boldsymbol{\alpha})}{\partial \alpha_j} \right] - c_j. \tag{T37}$$

Solving for α_j gives

$$\alpha_j^m = \frac{\sum_i \frac{\partial p_i}{\partial \alpha_j} \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) + \sum_i p_i(\boldsymbol{\alpha}) \cdot \left(T_{ij}(\boldsymbol{\alpha}) + \alpha_{-j} \cdot \frac{\partial T_{i,-j}}{\partial \alpha_j}(\boldsymbol{\alpha}) \right) - c_j}{- \sum_i p_i(\boldsymbol{\alpha}) \cdot \frac{\partial T_{ij}}{\partial \alpha_j}(\boldsymbol{\alpha})}. \tag{T38}$$

G.3.3 Separated Platform Solution with Coordinated Ad Delivery

The problem is

$$\max_{\alpha_j} \sum_{i \in \mathcal{U}_j} p_i(\boldsymbol{\alpha}) \cdot \alpha_j \cdot T_{ij}(\boldsymbol{\alpha}) - c_j \alpha_j. \tag{T39}$$

The first-order conditions are

$$0 = \sum_i \frac{\partial p_i}{\partial \alpha_j} \cdot \alpha_j \cdot T_{ij}(\boldsymbol{\alpha}) + p_i(\boldsymbol{\alpha}) \cdot T_{ij} + p_i(\boldsymbol{\alpha}) \cdot \alpha_j \cdot \frac{\partial T_{ij}}{\partial \alpha_j}(\boldsymbol{\alpha}) - c_j. \tag{T40}$$

Solving for α_j gives

$$\alpha_j^{s,c} = \frac{\sum_{i \in \mathcal{U}_j} \frac{\partial p_i}{\partial \alpha_j} \cdot \alpha_j \cdot T_{ij}(\boldsymbol{\alpha}) + \sum_{i \in \mathcal{U}_j} p_i(\boldsymbol{\alpha}) \cdot T_{ij} - c_j}{- \sum_{i \in \mathcal{U}_j} p_i(\boldsymbol{\alpha}) \cdot \frac{\partial T_{ij}}{\partial \alpha_j}(\boldsymbol{\alpha})}. \tag{T41}$$

This differs from equation (16) in the main text because rather than depending on platform-specific prices p_{ij}, it depends on an integrated price p_i. This means duplication loss does not impact ad load. The difference in incentives in the separated versus merged equilibrium still depends on user overlap, however. When all users are single-homers, the separated equilibrium is identical to the merged equilibrium. As the share of multi-homers increases, two differences emerge. First, as in Section G.1, the advertiser-side Cournot externality increases, which increases ad load in the separated equilibrium. Second, if platforms are substitutes, user diversion reduces the incentive to withhold ad load in the merged equilibrium relative to the separated equilibrium, which increases ad load in the merged equilibrium relative to the separated equilibrium. In general, ad load can be higher or lower than in the merged equilibrium, depending on overlap, user diversion, and price elasticity.

G.3.4 Separated Platform Solution with Uncoordinated Ad Delivery

The problem is

$$\max_{\alpha_j} \sum_{i \in \mathcal{U}_j} p_{ij}(\boldsymbol{\alpha}) \cdot \alpha_j \cdot T_{ij}(\boldsymbol{\alpha}) - c_j \alpha_j. \tag{T42}$$

Taking the first-order condition and solving for α_j gives

$$\alpha_j = \frac{\sum_{i \in \mathcal{U}_j} \frac{\partial p_{ij}}{\partial \alpha_j} \cdot \alpha_j \cdot T_{ij}(\boldsymbol{\alpha}) + \sum_{i \in \mathcal{U}_j} p_{ij}(\boldsymbol{\alpha}) \cdot T_{ij} - c_j}{-\sum_{i \in \mathcal{U}_j} p_{ij}(\boldsymbol{\alpha}) \cdot \frac{\partial T_{ij}}{\partial \alpha_j}(\boldsymbol{\alpha})}, \tag{T43}$$

where, assuming that $\frac{\partial \zeta_j}{\partial \alpha_j} \approx 0$,

$$\frac{\partial p_{ij}}{\partial \alpha_j} \approx -\left(1 - O'_{aj}\right) \cdot \frac{\eta \cdot \left(T_{ij} + \alpha_j \frac{\partial T_{ij}(\boldsymbol{\alpha})}{\partial \alpha_j}\right)}{Am} - \frac{p_{ij}\zeta_j}{\left(1 - \zeta_j O'_{aj}\right)} \frac{\partial O'_{aj}}{\partial \alpha_j}. \tag{T44}$$

The numerator in equation (T43) captures both the advertiser-side Cournot effect and the business stealing effect, both of which increase ad load relative to the merged equilibrium. The absence of $\frac{\partial T_{i,-j}}{\partial \alpha_j}$ in the numerator reflects the same user-diversion force expressed in the previous sub-section. This implies that ad load may be higher or lower than the combined equilibrium social optimum.

However, since the separated equilibrium now has inefficiently duplicated ad impressions, comparing ad load versus the social planner benchmark is not sufficient to make welfare judgements. For example, even if ad load were identical in the separated platform equilibrium with duplication as in Equation (T35), social welfare would be lower in the separated equilibrium because some ads would be inefficiently duplicated, lowering advertiser surplus. See Section 5 for details.

H Ad Duplication Experiment Appendix

H.1 Ad Selection and Ad Design

We developed 15 creatives for distinct products spanning five product categories. We selected products and categories to be representative of typical ads on Facebook and Instagram. To do so, we first picked five top product categories and the top three advertisers within each category by ad spending from the 2024 SensorTower Digital Market Index (SensorTower 2024). We identified each resulting advertiser's best-selling product and created ads that linked to pages promoting or allowing users to purchase that product.[21] To promote our ads, we created a "product picks" Facebook page for each product category. Such third-party product recommendations are allowed by Meta's terms of use.

Our five product categories were shopping, consumer packaged goods, media and entertainment, health and wellness, and food and dining. The Facebook pages used to promote products in each category were called, respectively, "The Shopping Spot," "Everyday Care Essentials," "Media Roundup," "Health and Wellness Essentials," and "Culinary Crave." These pages are public and viewable on Facebook.[22] Ad creatives used public-source advertising materials to approximate campaigns consumers would likely see. Where relevant, ads link to a site to purchase the advertised product, and otherwise link to a site describing the product in more detail.

Table T1 summarizes the companies, products, and creatives used within each category.

H.2 Experimental Design Details

For each ad creative, we first recruited 8 audiences to target in campaigns. To recruit audiences, we ran campaigns targeting US adults aged 18–65. To delineate audiences for retargeting, we used Meta's "Custom Audiences" feature. This feature allows advertisers to identify a set of users based on behaviors or characteristics, such as whether they have previously interacted with another ad or visited an advertisers' website. We used a feature that builds an audience based on users who view at least 25% of a video ad. We made video ads using the 3-second GIFs of the creatives displayed in Table T1, such that users for whom the ad is displayed for 0.75 seconds became part of an audience. For each audience, we excluded users from being targeted once they became part of any of the other audiences to ensure that the eight audiences recruited for each ad were non-overlapping. The campaigns used to recruit custom audiences were run over four days in January 2025, with a daily budget of $2.

[21] In August 2024, we used online sources to identify the top selling product for each company over the past month. When we could not identify clear top selling products, we identified a product recently the subject of a major ad campaign. We used some discretion to ensure the advertised product was associated with the target company – for example, although the top selling product on Amazon in July 2024 was the Stanley Cup water bottle, we selected the Amazon Fire TV stick (second most popular).

[22] The Facebook page IDs are: 61565619873336 (Shopping Spot), 61565078579057 (Everyday Care Essentials), 61564728493259 (Media Roundup), 61565433650990 (Health and Wellness Essentials), and 61565407492274 (Culinary Crave).

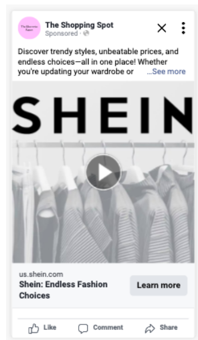
We then targeted these audiences with follow-up campaigns that ran for one week and started after the initial recruitment campaign began. Across audiences, we experimentally varied ad intensity, ad frequency, and campaign duplication. We designated 4 audiences to target with non-duplicated campaigns, and the remaining 4 to target with duplicated campaigns. Audiences assigned to the non-duplicated condition were targeted by one follow-up campaign. Audiences assigned to the duplicated condition were targeted by two identical follow-up campaigns to induce diminishing returns.

The four audiences within the non-duplicated and duplicated conditions were assigned to one of four treatments consisting of a daily campaign budget and campaign objective. These conditions were: (i) "low spend, clicks objective", (iii) "low spend, reach objective", (iv) "high spend, clicks objective", and (v) "high spend, reach objective." Low and high spend campaigns received a daily budget of \$2 and \$12 per day, respectively. Campaigns with a clicks objective were given the Meta performance goal of maximizing the number of link clicks, wheres campaigns with a reach objective were given the Meta performance goal of maximizing daily unique reach.

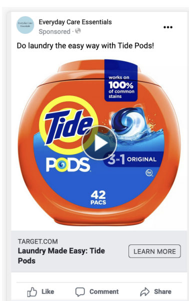
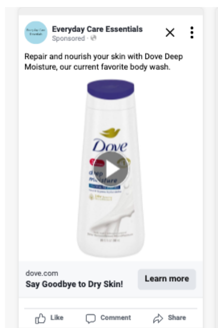
For each campaign, we measured the click-through rate, number of impressions, and campaign reach. We also gathered Meta's estimates of the combined unique reach of campaigns assigned to the duplicated condition, which, along with data on unique reach of each individual campaign, allows us to back out the fraction of a campaign audience that is impressed by both duplicated campaigns. We estimate campaign frequency as the ratio of number of impressions and reach, and follow-up campaign reach as the fraction of the initial audience impressed by the follow-up campaign.

Table T1: **Duplication Loss Experiment Creatives**

(a) **Shopping Ads**

Company	Amazon	Temu	Shein
Product	Fire Stick	Shopping	Shopping
Ad creative			

(b) **Consumer Packaged Goods Ads**

Company	Proctor & Gamble	Unilever	Nestle
Product	Tide Pods	Dove	Nescafe
Ad creative			

(c) **Media and Entertainment Ads**

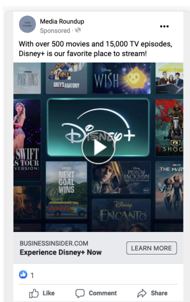
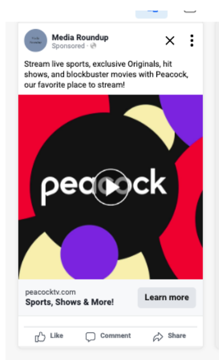
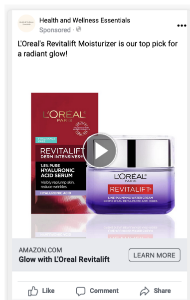
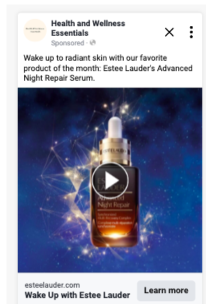
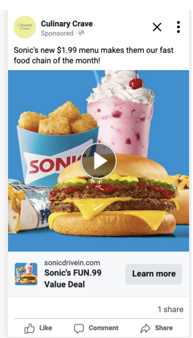
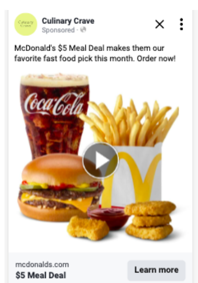
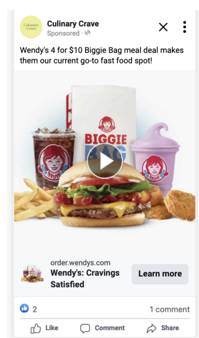
Company	Disney	NBC Universal	Amazon
Product	Disney+	Peacock	Amazon Prime
Ad creative			

Table T1: **Duplication Loss Experiment Creatives, Continued**

(d) **Health and Wellness Ads**

<table>
<tr><td>Company</td><td>L'Oreal</td><td>Estee Lauder</td><td>Olay</td></tr>
<tr><td>Product</td><td>Revitalift moisturizer</td><td>Night repair serum</td><td>Regenerist moisturizer</td></tr>
<tr><td>Ad creative</td>
<td>
</td>
<td>
</td>
<td>
</td></tr>
</table>

(e) **Food and Dining Services Ads**

<table>
<tr><td>Company</td><td>Sonic</td><td>McDonald's</td><td>Wendy's</td></tr>
<tr><td>Product</td><td>Sonic's $1.99 menu</td><td>$5 meal deal</td><td>Biggie Bag</td></tr>
<tr><td>Ad creative</td>
<td>
</td>
<td>
</td>
<td>
</td></tr>
</table>

Notes: This figure describes top companies and products used to develop ad creatives for the duplication experiment. Screen captures of ad creatives are for the video ads used to initially recruit custom audiences, as described in Section H.2, but are identical to the static ads used for follow-up campaigns.

I Structural Estimation Appendix

Throughout this appendix section, we use the notation M and N_j to refer to the number of multi-homers and users on platform j, respectively, satisfying $\mu_j = M/N_j$ and $M + \sum_j (N_j - M) = N$.

I.1 Average Time Use and FOCs as a Function of Time Use Moments

This appendix describes how to re-write equations (34) and (35) as functions of time use moments.

Define $e_{ij} := T_{ij} - T_{kj}$ as residual time use for group k, and $\mathcal{E}_{sj}^2$, $\mathcal{E}_{mj}^2$, and $\mathcal{E}_{m12}$ as the variance of e_{ij} for single-homers on j, the variance of e_{ij} for multi-homers on j, and $\mathcal{E}_{m12}$ as the covariance of e_{i1} and e_{m2} for multi-homers.

The modeled time-use weighted average of user-level ad prices p_i is given by:

$$\frac{\sum_{i\in\mathcal{U}} \boldsymbol{\alpha} \cdot \boldsymbol{T}_i\left(\boldsymbol{\alpha}^m\right) \cdot p_i}{\sum_{i\in\mathcal{U}} \boldsymbol{\alpha} \cdot \boldsymbol{T}_i\left(\boldsymbol{\alpha}^m\right)} = \frac{\sum_j \sum_{i\in\mathcal{U}_j} \alpha_j \cdot T_{ij}\left(\boldsymbol{\alpha}^m\right) \cdot p_i}{N \cdot \left(\alpha_1 T_1 + \alpha_2 T_2\right)} \tag{T45}$$

where the denominator follows because residual time use averages to zero. Since in the numerator T_{ij} is multiplied by p_i, which itself depends on $\boldsymbol{T}_i$, the numerator depends on residual time use variances and covariances. Specifically:

$$\sum_{i\in\mathcal{U}_j} \alpha_j T_{ij} p_i = \sum_{i\in\mathcal{U}_j} \eta \cdot \left(1 + \eta_0 - \frac{\boldsymbol{\alpha}\cdot\boldsymbol{T}_i}{Am}\right)\alpha_j T_{ij} \tag{T46}$$

$$= O \cdot \eta \cdot \left[\left(1+\eta_0\right)\alpha_j T_{mj} - \frac{\alpha_j^2\left(T_{mj}^2 + \mathcal{E}_{mj}^2\right) + \alpha_j\alpha_{-j}\left(T_{mj}T_{m,-j} + \mathcal{E}_{12}\right)}{Am}\right] +$$

$$\left(N_j - O\right)\cdot\eta\cdot\left[\left(1+\eta_0\right)\alpha_j T_{sj} - \frac{\alpha_j^2\left(T_{sj}^2 + \mathcal{E}_{sj}^2\right)}{Am}\right] \tag{T47}$$

The FOC on platform j is:

$$\sum_i \left[\frac{\partial p_i}{\partial \alpha_j}\cdot\boldsymbol{\alpha}\cdot\boldsymbol{T}_i\left(\boldsymbol{\alpha}\right) + p_i \cdot\left(T_{ij}\left(\boldsymbol{\alpha}\right) + \boldsymbol{\alpha}^m \cdot \frac{\partial \boldsymbol{T}_i\left(\boldsymbol{\alpha}\right)}{\partial\alpha_j}\right)\right] - c_j = 0 \tag{T48}$$

Rearranging slightly:

$$0 = \sum_i \frac{\partial \boldsymbol{\alpha}\cdot\boldsymbol{T_i}(\boldsymbol{\alpha})}{\partial\alpha_j}\cdot\left(p_i(\boldsymbol{\alpha}\cdot\boldsymbol{T_i}(\boldsymbol{\alpha}) + \boldsymbol{\alpha}\cdot\boldsymbol{T_i}(\boldsymbol{\alpha})\cdot\frac{dp_i}{d\boldsymbol{\alpha}\cdot\boldsymbol{T_i}(\boldsymbol{\alpha})}\right) - c_j \tag{T49}$$

$$= \sum_i \left[T_{ij}(\boldsymbol{\alpha}) + \alpha_j\frac{\partial T_{ij}(\boldsymbol{\alpha})}{\partial\alpha_j} + \alpha_{-j}\frac{\partial T_{i,-j}(\boldsymbol{\alpha})}{\partial\alpha_j}\right]\cdot\left[p_i(\boldsymbol{\alpha}\cdot\boldsymbol{T_i}(\boldsymbol{\alpha}) + \boldsymbol{\alpha}\cdot\boldsymbol{T_i}(\boldsymbol{\alpha})\cdot\frac{dp_i}{d\boldsymbol{\alpha}\cdot\boldsymbol{T_i}(\boldsymbol{\alpha})}\right] - c_j \tag{T50}$$

If i is a multi-homer:

$$\frac{\partial T_{ij}(\boldsymbol{\alpha})}{\partial \alpha_j} = -\frac{\gamma_j \sigma_{-j}}{\sigma_j \sigma_{-j} - \rho^2}, \quad \frac{\partial T_{i,-j}(\boldsymbol{\alpha})}{\partial \alpha_j} = -\frac{\gamma_j \rho}{\sigma_j \sigma_{-j} - \rho^2} \tag{T51}$$

If i is a single-homer:

$$\frac{\partial T_{ij}}{\partial \alpha_j} = \frac{-\gamma_j}{\sigma_j}, \quad \frac{\partial T_{i,-j}}{\partial \alpha} = 0 \tag{T52}$$

Moreover:

$$p_i(\boldsymbol{\alpha} \cdot \boldsymbol{T_i}(\boldsymbol{\alpha})) + \boldsymbol{\alpha} \cdot \boldsymbol{T_i}(\boldsymbol{\alpha}) \cdot \frac{dp_i}{d\boldsymbol{\alpha} \cdot \boldsymbol{T_i}(\boldsymbol{\alpha})} = \eta \left(1 + \eta_0 - \frac{\boldsymbol{\alpha} \cdot \boldsymbol{T_i}(\boldsymbol{\alpha})}{Am}\right) + \tag{T53}$$

$$\boldsymbol{\alpha} \cdot \boldsymbol{T_i}(\boldsymbol{\alpha}) \cdot \frac{\partial}{\partial \boldsymbol{\alpha} \cdot \boldsymbol{T_i}(\boldsymbol{\alpha})} \left[\eta(1 + \eta_0 - \frac{\boldsymbol{\alpha} \cdot \boldsymbol{T_i}(\boldsymbol{\alpha})}{Am}\right]$$

$$= \eta \left(1 + \eta_0 - \frac{\boldsymbol{\alpha} \cdot \boldsymbol{T_i}(\boldsymbol{\alpha})}{Am}\right) + \boldsymbol{\alpha} \cdot \boldsymbol{T_i}(\boldsymbol{\alpha}) \cdot \frac{-\eta}{Am} \tag{T54}$$

$$= \eta \left(1 + \eta_0 - 2\frac{\boldsymbol{\alpha} \cdot \boldsymbol{T_i}(\boldsymbol{\alpha})}{Am}\right) \tag{T55}$$

Substituting into equation (T50) and summing across multi-homers and single-homers, we have:

$$0 = FOC_m + \sum_j FOC_{sj} - c_j$$

where:

$$\begin{aligned} FOC_m = {} & M\eta \cdot (1 + \eta_0) \left[T_{mj} - \alpha_j \cdot \frac{\gamma_j \sigma_{-j}}{\sigma_j \sigma_{-j} - \rho^2} - \alpha_{-j} \cdot \frac{\gamma_j \rho}{\sigma_j \sigma_{-j} - \rho^2}\right] \\ & - \frac{2}{Am} M\eta \left[\alpha_j \cdot (T_{mj}^2 + \mathcal{E}_{mj}^2) + \alpha_{-j} \cdot (T_{mj} T_{m,-j} + \mathcal{E}_{m12}^2)\right] \\ & + \frac{2}{Am} \cdot M\eta \left[\left(\alpha_j^2 T_{mj} + \alpha_j \alpha_{-j} T_{m,-j}\right) \cdot \frac{\gamma_j \sigma_{-j}}{\sigma_j \sigma_{-j} - \rho^2} + \left(\alpha_{-j}^2 T_{m,-j} + \alpha_j \alpha_{-j} T_{mj}\right) \cdot \frac{\gamma_j \rho}{\sigma_j \sigma_{-j} - \rho^2}\right] \end{aligned} \tag{T56}$$

and

$$\begin{aligned} FOC_{sj} = {} & (N_j - M) \cdot \eta \cdot (1 + \eta_0) \cdot \left[T_{sj} - \alpha_j \cdot \frac{\gamma_j}{\sigma_j}\right] \\ & - \frac{2\eta}{Am} \cdot (N_j - M) \cdot \left[\alpha_j \cdot (T_{sj}^2 + \mathcal{E}_{sj}^2) - \alpha_j^2 T_{sj} \frac{\gamma_j}{\sigma_j}\right] \end{aligned} \tag{T57}$$

I.2 Standard Errors

The covariance matrix of $\boldsymbol{\Theta}$ is

$$\boldsymbol{\Sigma} = \mathbf{H}^{-1} \cdot \boldsymbol{\Omega}_h \cdot \mathbf{H}^{-1}, \tag{T58}$$

where H is

$$\mathbf{H} = \boldsymbol{R}'_\Theta W \boldsymbol{R}_\Theta. \tag{T59}$$

Because $\sqrt{n}(\hat{\boldsymbol{\delta}} - \boldsymbol{\delta}) \to_d N(0, \boldsymbol{\Omega}_\delta)$, according to delta method, $\boldsymbol{\Omega}_{\boldsymbol{h}}$ is

$$\sqrt{n}\boldsymbol{h}(\Theta_{D,}\pi) \to_d N(0, \boldsymbol{\Omega}_{\boldsymbol{h}}) = N(0, \boldsymbol{R}'_\Theta W \boldsymbol{R}_\delta \boldsymbol{\Omega}_\delta \boldsymbol{R}'_\delta W \boldsymbol{R}_\Theta) \tag{T60}$$

The matrices $\boldsymbol{R}_\Theta$ and $\boldsymbol{R}_\delta$ represent the Jacobian matrices with respect to the parameters $\boldsymbol{\Theta}$ to be estimated and the empirical moments $\boldsymbol{\delta}$:

$$\boldsymbol{R}_\Theta = \frac{\partial}{\partial \Theta_D} \boldsymbol{h}(\boldsymbol{\Theta}, \boldsymbol{\delta}) \tag{T61}$$

$$\boldsymbol{R}_\delta = \frac{\partial}{\partial \delta} \boldsymbol{h}(\boldsymbol{\Theta}, \boldsymbol{\delta}). \tag{T62}$$

A consistent estimator of Σ is

$$\hat{\boldsymbol{\Sigma}} = \hat{\mathbf{H}}^{-1} \cdot \hat{\boldsymbol{\Omega}}_h \cdot \hat{\mathbf{H}}^{-1}, \tag{T63}$$

where $\hat{\boldsymbol{\Omega}}_h$ is

$$\hat{\boldsymbol{\Omega}}_h = \hat{\boldsymbol{R}}'_\Theta \hat{W} \hat{\boldsymbol{R}}_\delta \hat{\boldsymbol{\Omega}}_\delta \hat{\boldsymbol{R}}'{}_\delta \hat{W} \hat{\boldsymbol{R}}_\Theta, \tag{T64}$$

$\hat{H}$ is

$$\hat{\mathbf{H}} = \hat{\boldsymbol{R}}'_\Theta \hat{W} \hat{\boldsymbol{R}}_\Theta, \tag{T65}$$

and $\hat{\boldsymbol{\Omega}}_\delta$ is

$$\hat{\boldsymbol{\Omega}}_\delta = \begin{pmatrix} \hat{\boldsymbol{\Omega}}_F & & & & & & \\ & \hat{\boldsymbol{\Omega}}_E & & & & & \\ & & \hat{\boldsymbol{\Omega}}_D & & & & \\ & & & \hat{\boldsymbol{\Omega}}_B & & & \\ & & & & \hat{\boldsymbol{\Omega}}_G & & \\ & & & & & \hat{\Omega}_P & \\ & & & & & & \hat{\boldsymbol{\Omega}}_L \end{pmatrix}. \tag{T66}$$

In the $\hat{\boldsymbol{\Omega}}_\delta$ matrix just above, $\hat{\boldsymbol{\Omega}}_F$ is the sample covariance matrix for $\left(\left\{\hat{T}^C_{kj}\right\}_{kj}\right)$ from FIES; $\hat{\boldsymbol{\Omega}}_E$ is the sample covariance matrix for $\left(\left\{\hat{\mathcal{E}}^2_{kj}\right\}_{j,k}, \widehat{\mathcal{E}}_{mFI}\right)$ estimated from FIES; $\hat{\boldsymbol{\Omega}}_D$ is the sample covariance matrix for $\left(\left\{\delta^j_{-j}\right\}_j\right)$ from FIES; $\hat{\boldsymbol{\Omega}}_B$ is the sample covariance matrix for $\left(\left\{\hat{T}^B_j\right\}_j, \left\{\hat{\tau}^C_j\right\}_j\right)$, estimated using Seemingly Unrelated Regressions in the DA data; $\hat{\boldsymbol{\Omega}}_G$ is the sample covariance matrix of $\left(\left\{\widehat{\frac{T_j^{Aj}-T_j^C}{T_j^C}}\right\}_j\right)$ from Brynjolfsson et al. (2024), $\hat{\Omega}_P$ is the variance of $\hat{p}^m$, and $\hat{\mathbf{\Omega}}_L$ gives the variance of $\hat{\lambda}$ from the duplication loss experiment. We calculate $\hat{\Omega}_P$ without assuming time-series draws of Birch (2025) prices follow the same distribution.[23]

We assume no covariance across different data sources—for example, between DA, the two different FIES experiments, Brynjolfsson et al. (2024), and our duplication loss experiment. Lacking micro-data for FIES diversion ratio estimates from Allcott, Kiefer, and Tangkitvanich (2025) and Brynjolfsson et al. (2024), we also assume $Cov(\hat{\delta}_F^I, \hat{\delta}_I^F) = Cov(\widehat{\frac{\Delta_\alpha T_F}{T_F^C}}, \widehat{\frac{\Delta_\alpha T_I}{T_I^C}}) = 0$and that there is no correlation between estimated diversion ratios, Control group time use, and residual time use variance. We expect the effect on standard errors to be small, since (i) we control for baseline time use in diversion ratio estimation, limiting potential correlation with average time use; and (ii) time use appears approximately jointly lognormal, in which case means and standard deviations are close to independent.

Since the estimator is just-identified, we use $\hat{W} = I$. We compute the Jacobians of $\boldsymbol{h}(\cdot)$ using MATLAB symbolic differentiation.

I.3 Sensitivity Matrix

Table T2 presents the Λ sensitivity matrix defined in Andrews, Gentzkow, and Shapiro (2017) for our estimator described in Section 4. In the notation of Appendix I.2, this is $-(\boldsymbol{R}_\Theta' \boldsymbol{I} \boldsymbol{R}_\Theta)^{-1} \boldsymbol{R}_\Theta' \boldsymbol{I}$. As described in Andrews, Gentzkow, and Shapiro (2017), each entry is the local sensitivity of parameter listed in the row to a small perturbation of the moment listed in the column. We divide each row by the parameter estimate and multiply each column by the empirical moment, so that each cell can be interpreted as the local elasticity of the parameter with respect to the moment. The Facebook and Instagram FOCs are not empirical moments, so we do not multiply those two columns by anything. Each cell in those two columns can be interpreted as the local percent change in the parameter in the row with respect to the moment in the column.

Table T3 presents a version of the matrix without multiplying by the empirical moment, so that each cell can be interpreted as the local percent change in the parameter in the row with respect to the moment in the column.

[23] Specifically, we assume that rather than observing $2T$ observations of p_t^m, we instead observe T draws of p_{tj}^m on each platform j, where p_{tj}^m have the same expectation but do not necessarily follow the same distribution. Then $\hat{\Omega}_p = \hat{\text{Var}}(\hat{p}^m) = \hat{\text{Var}}\left(\frac{\hat{p}_F^m + \hat{p}_I^m}{2}\right) = \frac{1}{4}\left(\hat{\text{Var}}(\hat{p}_F^m) + \hat{\text{Var}}(\hat{p}_I^m)\right) + \frac{1}{2}\hat{\text{Cov}}(\hat{p}_F^m, \hat{p}_I^m)$. We estimate each of these components using time-series variance and covariance for prices on each platform.

Table T2: **Sensitivity Matrix in Elasticity Units**

	Single-homer average FB use	Multi-homer average FB use	Single-homer average IG use	Multi-homer average IG use	Average diversion ratio	FB bonus response	IG bonus response	Effect of FB ad removal	Effect of IG ad removal	Duplication effect on ln(CTR)	Average ad price	FB FOC	IG FOC
ξ_{sF}	0.505	-0.485	-0.005	-0.015	-0.024	-0.980	-0.020	0.083	0	0	0	0	0
ξ_{mF}	-0.405	0.392	-0.002	0.015	0.015	-0.973	-0.027	0.084	0	0	0	0	0
ξ_{sI}	-0.023	-0.005	0.791	-0.763	-0.129	-0.028	-0.972	0	0.074	0	0	0	0
ξ_{mI}	-0.056	0.056	-0.107	0.108	0.005	-0.124	-0.876	0	0.076	0	0	0	0
ρ	-0.290	-0.518	0.080	-0.271	0.958	-0.808	-0.192	0	0	0	0	0	0
σ_F	-0.450	-0.530	-0.005	-0.015	-0.023	-0.980	-0.020	0	0	0	0	0	0
σ_I	-0.023	-0.005	-0.145	-0.827	-0.128	-0.028	-0.972	0	0	0	0	0	0
γ_F	0.008	0.015	-0.006	-0.016	-0.031	-0.978	-0.022	1.00	0	0	0	0	0
γ_I	-0.023	-0.002	-0.012	0.037	-0.141	-0.025	-0.975	0	1.00	0	0	0	0
κ	0.225	0.256	0.005	0.004	0.001	-0.001	0.001	-0.050	0	-1.11	0.350	-84.5	0
η	0.590	0.702	-0.008	-0.099	0.005	-0.004	0.004	-0.221	0	0	2.28	-377	0
η_0	0.115	0.124	-0.001	-0.034	0.001	-0.001	0.001	-0.052	0	0	0.365	-88.1	0
c_I	-0.176	-1.17	0.317	1.15	0.019	0.006	-0.006	0.153	-0.142	0	0.085	262	-1349

Notes: This table presents the sensitivity matrix defined in Andrews, Gentzkow, and Shapiro (2017) for our estimator described in Section 4. We divide each row by the parameter estimate and multiply each column by the empirical moment, so that each cell can be interpreted as the local elasticity of the parameter in the row with respect to the moment in the column. The Facebook and Instagram FOCs are not empirical moments, so we do not multiply those two columns by anything. Each cell in those two columns can be interpreted as the local percent change in the parameter in the row with respect to the moment in the column.

Table T3: **Sensitivity Matrix in Semi-Elasticity Units**

	Single-homer average FB use	Multi-homer average FB use	Single-homer average IG use	Multi-homer average IG use	Average diversion ratio	FB bonus response	IG bonus response	Effect of FB ad removal	Effect of IG ad removal	Duplication effect on ln(CTR)	Average ad price	FB FOC	IG FOC
ξ_{sF}	0.701	-0.708	-0.020	-0.067	-0.316	2.35	0.049	0.888	0	0	0	0	0
ξ_{mF}	-0.562	0.572	-0.007	0.067	0.207	2.34	0.066	0.893	0	0	0	0	0
ξ_{sI}	-0.033	-0.007	3.01	-3.46	-1.72	0.067	2.37	0	0.788	0	0	0	0
ξ_{mI}	-0.078	0.081	-0.406	0.488	0.074	0.299	2.13	0	0.813	0	0	0	0
ρ	-0.403	-0.756	0.303	-1.23	12.8	1.94	0.467	0	0	0	0	0	0
σ_F	-0.624	-0.774	-0.020	-0.067	-0.307	2.35	0.048	0	0	0	0	0	0
σ_I	-0.033	-0.007	-0.550	-3.75	-1.71	0.068	2.37	0	0	0	0	0	0
γ_F	0.011	0.021	-0.024	-0.072	-0.416	2.35	0.054	10.6	0	0	0	0	0
γ_I	-0.032	-0.003	-0.046	0.167	-1.88	0.059	2.38	0	10.6	0	0	0	0
κ	0.313	0.374	0.017	0.018	0.016	0.002	-0.002	-0.527	0	3.32	34.4	-84.5	0
η	0.820	1.02	-0.029	-0.449	0.072	0.009	-0.009	-2.35	0	0	224	-377	0
η_0	0.159	0.181	-0.004	-0.153	0.017	0.002	-0.002	-0.549	0	0	35.9	-88.1	0
c_I	-0.245	-1.71	1.21	5.21	0.254	-0.016	0.016	1.63	-1.51	0	8.33	262	-1349

Notes: This table presents the sensitivity matrix defined in Andrews, Gentzkow, and Shapiro (2017) for our estimator described in Section 4. We divide each row by the parameter estimate, so that each cell can be interpreted as the local percent change in the parameter in the row with respect to the moment in the column.

I.4 Aggregate Elasticity of Ad Demand in Merged Equilibrium

We define the aggregate elasticity of ad demand on platform j as

$$\varepsilon_j^D := \frac{\partial Q_j}{\partial P_j} \cdot \frac{P_j}{Q_j}, \tag{T67}$$

where

$$P_j := \sum_{i \in \mathcal{U}_j} T_{ij} \cdot p_i, \quad q_i := Am \cdot \left(1 - \frac{p_i}{\eta} + \eta_0\right), \quad Q_j := \sum_{i \in \mathcal{U}_j} T_{ij} \cdot q_i. \tag{T68}$$

The aggregate elasticity of ad demand is the elasticity of time-weighted ad demand, given by Q_j, with respect to time-weighted ad price, P_j. Since

$$\begin{aligned} Q_j &= \sum_{i \in \mathcal{U}_j} T_{ij} \cdot Am - \frac{Am}{\eta} T_{ij} \cdot p_i + T_{ij} \cdot Am \cdot \eta_0 & \text{(T69)} \\ &= \sum_{i \in \mathcal{U}_j} T_{ij} \cdot Am \cdot (1 + \eta_0) - \frac{Am}{\eta} P_j, & \text{(T70)} \end{aligned}$$

we have

$$\varepsilon_j^D = -\frac{Am}{\eta} \frac{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot p_i}{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot q_i}. \tag{T71}$$

We define the elasticity in this way to preserve intuitive properties of the elasticity in the merged equilibrium – namely, that a monopolist sets an aggregate elasticity of demand on each platform weakly above one, strictly so if either costs are positive or users are averse to ad load. To illustrate this property, note the merged equilibrium problem is:

$$\max_{\boldsymbol{\alpha}} \sum_i p_i(\boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha})) \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) - \boldsymbol{c} \cdot \boldsymbol{\alpha} \tag{T72}$$

In this notation, the FOC with respect to α_j is

$$\begin{aligned} c_j &= \sum_i \frac{\partial}{\partial \alpha_j} (\boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha})) \cdot \left(p_i + \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot \frac{\partial p_i}{\partial (\boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}))}\right) \\ &= \sum_i \left(T_{ij}(\boldsymbol{\alpha}) + \boldsymbol{\alpha} \cdot \frac{\partial \boldsymbol{T}_i}{\partial \alpha_j}\right) \cdot \left(p_i - \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot \frac{\eta}{Am}\right). & \text{(T73)} \end{aligned}$$

Rearranging gives

$$c_j - \sum_i \boldsymbol{\alpha} \cdot \frac{\partial \boldsymbol{T}_i}{\partial \alpha_j} \cdot \left(p_i - \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot \frac{\eta}{Am}\right) = \sum_i T_{ij}(\boldsymbol{\alpha}) \cdot \left(p_i - \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot \frac{\eta}{Am}\right). \tag{T74}$$

This illustrates the typical two-sided market intuition that the merged platform balances the elasticity of time use (LHS) with the elasticity of demand from advertisers (RHS). Define the effective cost of higher ad load as $\tilde{c}_j := c_j - \sum_i \boldsymbol{\alpha} \cdot \frac{\partial \boldsymbol{T}_i}{\partial \alpha_j} \cdot \left(p_i - \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot \frac{\eta}{Am}\right)$, which accounts for both "physical" costs from higher ad load as well costs from lost infra-marginal revenue due to lower time use. Substitute $q_i = \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha})$. Rearranging equation (T74) gives

$$\frac{\tilde{c}_j - \sum_{i \in \mathcal{U}_j} T_{ij}(\boldsymbol{\alpha}) \cdot p_i}{\sum_{i \in \mathcal{U}_j} T_{ij}(\boldsymbol{\alpha}) \cdot p_i} = -\frac{\eta}{Am} \cdot \frac{\sum_{i \in \mathcal{U}_j} T_{ij}(\boldsymbol{\alpha}) \cdot q_i}{\sum_{i \in \mathcal{U}_j} T_{ij}(\boldsymbol{\alpha}) \cdot p_i} \iff \frac{P_j - \tilde{c}_j}{P_j} = -\frac{1}{\varepsilon_j^D}. \tag{T75}$$

This shows that under our definition of ε_j^D, merged equilibrium ad load follows an analogy to the inverse elasticity markup rule. This implies that $\varepsilon_j^D \leq -1$, with greater absolute value indicating higher effective costs.

To compute ε_j^D, the numerator is the same as equation (T78), divided through by α_j. The denominator is

$$\begin{aligned} \sum_i \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot T_{ij}(\boldsymbol{\alpha}) = M \cdot \left[\alpha_j \cdot \left(T_{mj}^2 + \mathcal{E}_{mj}^2\right) + \alpha_{j'} \cdot \left(T_{mj} T_{mj'} + \mathcal{E}_{12}\right)\right] \\ + (N_j - M) \cdot \left[\alpha_j \cdot \left(T_{sj}^2 + \mathcal{E}_{sj}^2\right)\right]. \end{aligned} \tag{T76}$$

J Miscellaneous Formulas

J.0.1 Average Price per Impression

Average price per actual impression on platform j is:

$$\overline{p}_j = \frac{\sum_{i \in \mathcal{U}_j} p_{ij} \cdot \alpha_j T_{ij}}{\sum_{i \in \mathcal{U}_j} \alpha_j T_{ij}} \tag{T77}$$

The denominator is straightforward to calculate. In the merged equilibrium, the numerator is:

$$\begin{aligned} \sum_{i \in \mathcal{U}_j} p_i \alpha_j T_{ij} &= \sum_{i \in \mathcal{U}_j} \eta \cdot \left(1 + \eta_0 - \frac{\boldsymbol{\alpha} \cdot \boldsymbol{T}_i}{Am}\right) \alpha_j T_{ij} \\ &= M \cdot \eta \cdot \left[\left(1 + \eta_0\right) \alpha_j T_{mj} - \frac{\alpha_j^2 \left(T_{mj}^2 + \mathcal{E}_{mj}^2\right) + \alpha_j \alpha_{-j} \left(T_{mj} T_{m,-j} + \mathcal{E}_{12}\right)}{Am}\right] \\ &+ (N_j - M) \cdot \eta \cdot \left[\left(1 + \eta_0\right) \alpha_j T_{sj} - \frac{\alpha_j^2 \left(T_{sj}^2 + \mathcal{E}_{sj}^2\right)}{Am}\right] \end{aligned} \tag{T78}$$

In the separated equilibrium, the numerator is:

$$\sum_{i\in\mathcal{U}_j} p_{ij}\alpha_j T_{ij} = (1-\zeta O'_{aj})\cdot\eta\cdot\sum_{i\in\mathcal{U}_j}\left(1+\eta_0-\frac{\alpha_j T_{ij}}{Am}\right)\alpha_j T_{ij}$$
$$= (1-\zeta O'_{aj})\cdot\eta\cdot M\cdot\left((1+\eta_0)\,\alpha_j T_{mj}-\frac{\alpha_j^2\left(T_{mj}^2+\mathcal{E}_{mj}^2\right)}{Am}\right)$$
$$+ (1-\zeta O'_{aj})\cdot\eta\cdot(N_j-M)\cdot\left((1+\eta_0)\,\alpha_j T_{sj}-\frac{\alpha_j^2\left(T_{sj}^2+\mathcal{E}_{sj}^2\right)}{Am}\right) \tag{T79}$$

J.0.2 Formula for $\zeta_j(\boldsymbol{\alpha})$

By definition:

$$\zeta_j(\boldsymbol{\alpha}) = 1-\frac{\kappa}{\eta\cdot(1+\eta_0)}\frac{E_i\left[T_{ij}(\boldsymbol{\alpha})\cdot p_i^m | i\in\mathcal{U}_m\right]}{T_{mj}(\boldsymbol{\alpha})} \tag{T80}$$

Since:

$$E_i\left[T_{ij}(\boldsymbol{\alpha})\cdot p_i^m | i\in\mathcal{U}_m\right] = M^{-1}\sum_{i\in\mathcal{U}_m} T_{ij}(\boldsymbol{\alpha})\cdot p_i^m \tag{T81}$$
$$M^{-1}\sum_{i\in\mathcal{U}_m}\left[\eta\cdot\left(1+\eta_0-\frac{\boldsymbol{\alpha}^m\cdot\boldsymbol{T}_i(\boldsymbol{\alpha}^m)}{Am}\right)\cdot T_{ij}(\boldsymbol{\alpha})\right] \tag{T82}$$
$$= \eta\cdot\left[(1+\eta_0)\cdot T_{mj}(\boldsymbol{\alpha})-\frac{\alpha_j^m\left(T_{mj}(\boldsymbol{\alpha}^m)T_{mj}(\boldsymbol{\alpha})+\mathcal{E}_{mj}^2\right)+\alpha_{-j}^m\left(T_{m,-j}(\boldsymbol{\alpha}^m)T_{mj}(\boldsymbol{\alpha})+\mathcal{E}_{12}\right.}{Am}\right. \tag{T83}$$

the formula for $\zeta_j(\boldsymbol{\alpha})$ is

$$\zeta_j(\boldsymbol{\alpha}) = 1-\frac{\kappa}{1+\eta_0}\cdot T_{mj}^{-1}(\boldsymbol{\alpha})\cdot\left((1+\eta_0)\cdot T_{mj}(\boldsymbol{\alpha})-\frac{\alpha_j^m\left(T_{mj}(\boldsymbol{\alpha}^m)T_{mj}(\boldsymbol{\alpha})+\mathcal{E}_{mj}^2\right)+\alpha_{-j}^m\left(T_{m,-j}(\boldsymbol{\alpha}^m)T_{mj}(\boldsymbol{\alpha})+\mathcal{E}_{12}\right.}{Am}\right. \tag{T84}$$